FUZZY REGULATION MODEL OF THE INTERACTION BETWEEN THE STATE AND ECONOMIC SUBJECT WITHIN THE SHADOW ECONOMY AND TAX FIELD

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Abstract. State as an instance of tax authority and economic subject have aims respectively with different directions as tax collection and accomplishment of tax responsibilities. At the same time each of them makes effort to somehow benefit as a result of their own activity. If the utility of the state is expressed by the amount of taxes collected to the state budget, then the utility of the economic subject can be expressed with the left amount of its revenue after accomplishment of tax responsibility.

On the other hand, if we consider that state is socially responsible for the sustainability of the activity of economic subject, then another component of benefit of the state becomes obvious and this is reflected in the left revenue of the economic subject.

At the same time economic subject also owns a utility function of two components. Economic subject should both try to save the left revenue for its social security and accomplish the commitment of corporate responsibility to the state budget.

Thus, both them possess utility function depending on the left revenue and tax amount paid to the state budget. At the same time state should control truthfulness of the tax base of economic subject and the existence of its risk for the emergence of shadow economy. It is clearly seen that within the process of investigation of interaction between state and economic subject a number of economic and social factors should be taken into consideration. In this case investigation of the emergence of shadow economy and the interaction of its measurement with other economic factors is not an exception, either. In this article economic system is reviewed as an active system; and evaluation and reduction of the process emergence rate of shadow economy via optimization methods is being analyzed within economic relations between the state as a tax authority and economic subject as a taxpayer.

In this case, profitability of economic activity as a main influencing factor, expenses for staying in the shadow, contribution of tax and revenue for general utility are evaluated through fuzzy approach. Selecting profitability as a main factor is related with the process of not declaring the real level of profitability to tax authority by economic subject.

Keywords: State and economic subject, shadow economy, optimization, profitability, tax evasion, mathematical model, fuzzy approach

The reasons of the emergence of shadow economy and its assessment methods

Currently there are numerous approaches to the forms of emergence of shadow economy, the reasons for this and its assessment methods (1-5). Different aspects of the considered problem are reflected in these approaches and within related models, but their universality level is far away from the perfection. Some reasons for this are substantiated within scientific literature through wide variety of research papers (6-8). Main consequence reflected here explains that considering tax as only a way of filling the state budget is not satisfactory. Taxes are the tools for positive and negative impact on economic processes formed between the state and economic subject. That is why in order to create a model reflecting activity of both economic subject and supervisory authority economic factors allowing the research on emergence process of shadow economy while allocating resources between them and; the ones paving the

way for competitive environment which can help to find factors influencing the emergence of shadow economy should be preferred (8,9).

The behavioural model of the state

The utilization function of the state depends on the left revenue after paying all the taxes and overall amount of taxes paid to the state budget; and it is written as following:

$$D = \beta \ln[G(1 - u)r_0 + G(r - r_0)] + \alpha \ln Gur_0,$$

Here,

r – refers to real profitability;

r_0 – refers to declared profitability of the economic subject;

Gr_0 – refers to revenue;

$G(r - r_0)$ – refers to revenue left in the shadow;

$G(1 - u)r_0 + G(r - r_0)$ - refers to the revenue of tax payer;

Gur_0 – refers to the amount of tax receipts;

α – refers to the weight of tax receipts in the utility function;

β – refers to the special weight of the revenue of tax payer in the utility function after tax payments.

In this case if we consider P to be the forecast task on tax receipts mathematical model of the state as a tax authority can be written as following:

$$D = \beta \ln[G(1 - u)r_0 + G(r - r_0)] + \alpha \ln Gur_0 \rightarrow \max \quad (1.1)$$

$$Gur_0 + G\,u(r - r_0)(1 - k) \geq P; \quad (1.2)$$

$$0 \leq r_0 \leq r\ ;\ \alpha, \beta, u \in (0,1)\ ;\ \alpha + \beta = 1;\ \alpha, \beta, k \geq 0 \quad (1.3)$$

Here, k refers to the costs for staying in the shadow (or penalty for tax evasion).

Restriction condition (1.2) explains that economically the amount of tax receipts cannot be less than the amount of forecast task. (1.3) refers to the necessary restriction conditions.

Mathematical model of the economic subject

Defining and fulfilling the tax liability is not less important than the efforts of the economic subject for getting maximum revenue which seems to be the main aim of its activity. In most cases the amount which is separated and paid to the state budget is considered to be a lost and attempts for decreasing this amount are made both via legal and illegal methods by the economic subject. As a result some or whole amount of revenue can be hidden this way.

And this activity reflects in the declaration of economic subject in the way it knows. For this reason, utility gained regarding this activity consists of two parts: first part is the one declared and all related tax paid, the other part occurs to be the hidden one in which tax liability is not accomplished. It becomes obvious that utility function of the economic subject should include both parts. We can write following referring to afore given legend for the previous formula:

$$S = \beta \ln[G(1 - r_0)u + G(r - r_0)\,(1 - k)] + \alpha \ln Gur_0,$$

Here,

$G(1 - r_0)u + G(r - r_0)\,(1 - k)$ – refers to the revenue of the economic subject;

It is clear that the amount of tax payment of each economic subject can not exceed its tax potential, and declared profitability can not be larger than real profitability. Tax payer makes effort to maximize its utility within these conditions.
In this case if consider the tax potential of the tax payer as VP, we can write the mathematical model of the economic subject as following:

$$S = \beta \ln[G(1-u)r_0 + G(r-r_0)(1-k)] + \alpha \ln Gur_0 \to \max \quad (2.1)$$

$$Gur_0 + G\,u(r-r_0)(1-k) \le VP; \quad (2.2)$$

$$0 \le r_0 \le r\ ;\ \alpha, \beta, u \in (0,1)\ ;\ \alpha + \beta = 1;\ \alpha, \beta, k \ge 0 \quad (2.3)$$

Here restriction condition (2.2) explains that tax payments of the economic subject can not exceed its tax potential. (2.3) refers to necessary restriction conditions.
We should mention that as economic subject is inclined to stay in the shadow its α, β contribution to general utility changes in dependence of k – which refers to the costs for staying in the shadow.

Establishing IF-THEN fuzzy model

As mentioned above in the first stage the economic subject declares some part(r_0) of its real profitability (r) and misappropriates the left part($r - r_0$). Tax authority is aware about the r as they know the average price level of the field in which economic subject operate. That is why tax authority defines k – fine to eliminate this suspense within the report of the economic subject and informs economic subject about it. Economic subject takes the fine rate into account and in order not to lose the revenue declares a new and higher price, etc.
Let us analyse the evaluation of r_0 as profitability of economic activity playing the role of main factor, k as cost for staying in the shadow, α, β contribution of tax and revenue to general utility with the help of IF-THEN fuzzy approach (10,12). For this, in accordance with tax potential of economic subject, considering the conditions given in (1.3) and (2.3), to ease the formula, let us assume that $\beta = k$ and $\alpha = 1 - \beta$; to find appropriate rate for k we can present fuzzy approach as given below:
-let us exaggerate k and r_0 via fuzzification methods:
- let us define linguistic quantity for the rates of k and α:
“weak”, “average”, “strong”
We should also express the rates of r_0 with three linguistic quantity:
“low”, “average”, “high”
In accordance with these linguistic variables IF-THEN conditions can be presented as following:
IF k = "weak" and α = "strong" Then r_0 = "high"
IF k = "average" and α = "average" Then r_0 = "average"
IF k = "strong" and α = "weak" Then r_0 = "low"
IF k = "strong" and α = "strong" Then r_0 = "high"
IF k = "weak" and α = "weak" Then r_0 = "low"
Fragment from calculations of suggested model and fuzzy approach is given in an example below:
Initial data: let us assume that, G=4000$, P=150$, VP=500$,
$u = 0.2 \quad r = 0.33, \quad r_{min} = 0.21, \quad k = 0.2$

The growth in the utility function of state depending on the increased declared profitability of the economic subject is provided within the graph given below:

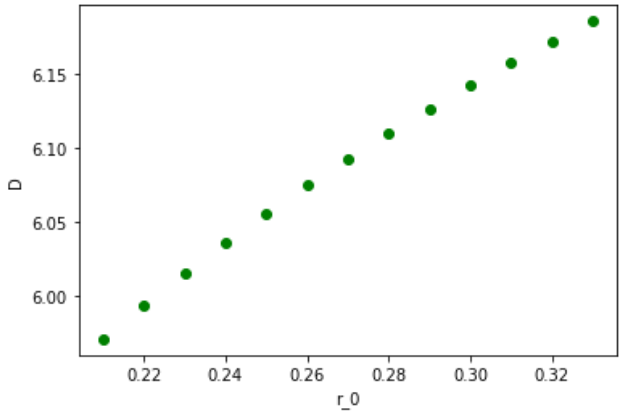

Fig. 1. The rate of utility function of state (D) increases in accordance with the growth of r_0

While the cost for staying in the shadow is increasing utility function of the state decreases. For $k = 0.4$ case change in rate variation depending on its declared profitability is shown in the below given graph:

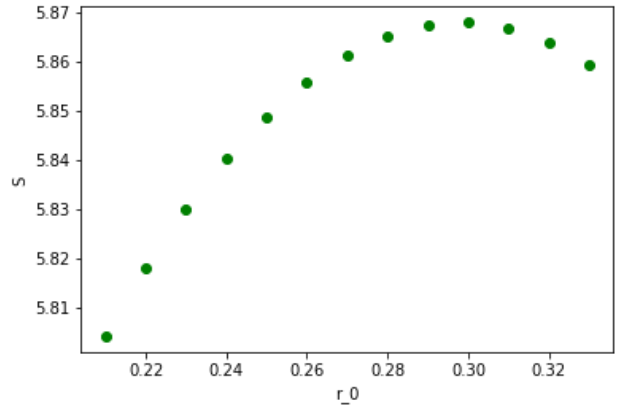

Fig. 2. Variation in the rate of utility function of an economic subject (S) depending on r_0

As seen from the picture, the rate of fine for staying in the shadow reaches to a such level that afterwards staying in the shadow does not seem to be beneficial, it even increases the costs.

CONCLUSION

- Achieved consequences show that suggested instrumentation reflects the emergence of shadow economy real enough;
- While analysing these graphs it is easily observed that k – increased fine rate respectively increases the inclination for leaving the shadow economy of economic subject;

- As declared profitability rate r_0 becomes similar to real profitability rate r the tax receipts of the state budget increases (Figure 1).
- In order not to have decreased rate for utility function economic subject is being forced to declare higher rates of r_0 to the state (Figure 2).
- In some rate of k-fine for staying in the shadow, the rate of r_0 becomes extremely closer to rate of real profitability r .
- While rate for fine k is increasing the shadow revenue noticeably decreases.

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