Fuzzy model of estimating the specific weight of shadow economy being a manifestation of economic crime on the basis of the utility function

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Abstract. The article is devoted to the control measures of the tax authority against the cases of tax reduction that occur during the fulfillment of the tax liability of the economic entity and the mechanism for detecting tax evasion. On this basis, in the assessment of quantitative and qualitative indicators, using the methodology of fuzzy logic IF − THEN, it is possible to increase the declared income only by increasing the fines without paying attention to the usefulness of the collection, or by increasing the usefulness of the collection without changing the fines. The article assumed that declared profitability, specific weight of the collected tax and the specific weight of income remaining at the disposal of a taxpayer were taken as fuzzy variables. However, the analysis shows that profitability differs both by fields and by taxpayers operating in each field. Therefore, the declared profitability factor and the penalty for its concealment should be treated differently.

Keywords: Shadow Economy, Economic Crime, Specific Weight of Tax, Fuzzy Optimization Mechanism, Fuzzy Approach

1   Introduction

The shadow economy is one of the main determinants of crime in the field of economic activity, especially its more organized manifestation forms. Therefore, ensuring the effective struggle against the "shadow economy" is considered one of the main directions of state policy of the Republic of Azerbaijan. Reforms carried out in this area in recent years, complex institutional measures have significantly reduced the characteristics of the "shadow economy", in particular, its widespread informal

employment, conducting bookkeeping by double-entry. However, due to the large potential of the "shadow economy", the effective struggle against it remains its significance in terms of increasing state budget revenues [1].

The phenomenon of shadow economy is one of the problems forthcoming in modern economics, which requires scientific estimation and practical solutions. Analysis of the emergence process of this problem and review of existing scientific research shows that one of the main reasons affecting its emergence is related to the gaps in tax policy, inadequacy of tax legislation to economic policy, inadequate stimulation of entrepreneurship, etc.

The fuzzy model of the interaction between the two components of economic system as an economic entity in the example of the state and taxpayer in the form of a tax authority, the process of shadow economy emergence in this economic relationship and reducing its specific weight was proposed [2]. The article assumed that declared profitability, specific weight of the collected tax and the specific weight of income remaining at the disposal of a taxpayer were taken as fuzzy variables. However, the analysis shows that profitability differs both by fields and by taxpayers operating in each field. Therefore, the declared profitability factor and the penalty for its concealment should be treated differently.

2   The mechanism of joint action of tax authority and economic entity

One of the main tasks of the tax authority, which represents the state and communicates directly with economic entities within the powers granted to it by legislation, by controlling their timely and full fulfillment of tax obligations, is to prevent tax evasion. Here, the state control function consists of cameral, mobile and operational inspections. As it is seen, the state (tax authority) and economic entity operate in a two-tier hierarchical relationship in the process of fulfilling the tax liability [3]. At the same time, state and economic entities involved in this relationship are characterized by certain, and sometimes significant, differences in corporate goals.

Each of them tries to get some benefit as a result of their activities. If in this case the profit of the state is expressed by the amount of tax mobilized to the state budget, the profit of the economic entity is expressed by the amount of income remaining at its disposal after fulfillment of the tax obligation [4].

On the other hand, if we consider that the state is socially responsible to the economic entity for the sustainability of its activities, then the second component of the state profit will appear, which is reflected in the income remaining at the disposal of the economic entity.

At the same time, the economic entity has a two-component utility function. In addition to trying to keep the income at its disposal for its social security, the economic entity should make appropriate allocations to the state budget in accordance with its corporate responsibility and legislation [5].

Thus, both of them have utility function, which depends on the disposable income and taxes paid to the state budget. In [6], A.B. Kostin calls the income remaining at the

disposal of the economic entity as its consumption, and assumes that the utility function for both entities is Cobb-Douglas shaped. Note that in this study, we will also use this hypothesis.

At the same time, the state should also take control measures on the accuracy of the taxation base of the economic entity, the existence risk of shadow economy, and so on. As it is seen, it is very important to take into account numerous economic and social indicators in the process of studying the relationship between the state and economic entities. Here, the emergence of shadow economy and the study of the relationship of its measurement with other economic indicators is no exception.

3   The utility function of the state

The utility function of the state is written as follows:

$$Y_1 = A * V^\alpha * C^\beta \to \max_{r_0} \qquad (1)$$

Where,
Y_1 - is the overall utility function of the state;
V - is tax revenue;
A - positive constant
C - is income remaining at the disposal of the economic entity;
r_0 - is the level of profitability declared by the economic entity;
α, β - accordingly, is the measure of the contribution of the tax mobilized to the state budget and the income remaining at the disposal of the economic entity and acting as a guarantee for its consumption to the total utility.

It is obvious that the income to the state budget is expressed in the tax calculated from the income determined in accordance with the declared profitability of the economic entity with the application of the tax rate fixed by the tax legislation:

$$V = G * u * r_0 \qquad (2)$$

Where,
G - is the value of the assets declared by the economic entity;
$G * r_0$ - is the income declared by the economic entity;
u - is the tax rate.

The income (consumption) remaining at the disposal of the economic entity C will be calculated as follows:

$$C = (1\text{-}u) * G * r_0 + G*(r\text{-}r_0) \qquad (3)$$

Where,
r - is real profitability;
$r - r_0$ — is profitability kept in the shadow;
$G*(r\text{-}r_0)$ - is the income remaining in the shadow.

As it is seen, the income remaining at the disposal of an economic entity consists of two parts:

$(1-u) * G * r_0$ - is income remaining after payment of tax calculated on declared income;

$G * (r-r_0)$ - is income hidden in full.

Thus, the logarithmic line of the utility function of the state can be

$$\ln Y_1 = \ln A + \alpha \ln V + \beta \ln C \qquad (4)$$

or

$$D = \ln A + \alpha \ln(G * u * r_0) + \beta \ln[(1-u) * G * r_0 + G * (r-r_0)] \qquad (5)$$

Where, $D = \ln Y_1$.

In this case, if we indicate the forecast task on tax revenues by P, then we can write the mathematical model of the state in the example of the tax authority as follows:

$$D = \ln A + \beta \ln[G(1-u)r_0 + G(r-r_0)] + \alpha \ln G u r_0 \rightarrow \max \qquad (6)$$

$$G u r_0 + G u(r-r_0)(1-k) \geq P; \qquad (7)$$

$$0 \leq r_0 \leq r ; \ \alpha, \beta, u \in (0,1) ; \ \alpha + \beta = 1; \ \alpha, \beta, k \geq 0 \qquad (8)$$

Where, k - is the income remaining in the shadow (penalty for tax evasion)

The constraint indicates that the conditionally economically collected tax cannot be less than the tax forecast task. (8) indicates the necessary constraint conditions.

4   Utility function of the economic entity

The utility function of the economic entity is also Cobb-Douglas function shaped:

$$Y_2 = B * V^\alpha * C^\beta \rightarrow \max_{r_0}$$

Where,

Y_2 - is utility function of the economic entity;

V - is the amount of tax paid by the economic entity;

C - is income remaining at the disposal of the economic entity.

α, β - accordingly, is the measure of the contribution of the tax mobilized to the state budget and the income remaining at the disposal of the economic entity and acting as a guarantee for its consumption to the total utility.

The tax paid by the economic entity to the state budget will be expressed as the tax calculated from the income determined in accordance with its declared profitability with the application of the tax rate fixed by the tax legislation:

$$V = G * u * r_0 \qquad (9)$$

The income (consumption) remaining at the disposal of the economic entity C will be calculated as follows:

$$C = (1-u) * G * r_0 + G * (r-r_0) * (1-k) \qquad (10)$$

Where,

k - is the penalty imposed by the state for the economic entity tending to remain in the shadow (the cost remaining in the shadow).

Thus, the logarithmic line of the utility function of the economic entity will be

$$\ln Y_2 = \ln B + \alpha \ln V + \beta \ln C \qquad (11)$$

or

$$S = \ln B + \alpha \ln(G * u * r_0) + \beta \ln[(1 - u) * G * r_0 + G * (r - r_0) * (1 - k)] \qquad (12)$$

Where, $S = \ln Y_2$

In this case, if we indicate the tax potential of the taxpayer by VP, then we can write the mathematical model of the economic entity as follows:

$$S = \beta \ln[G(1 - u)r_0 + G(r - r_0)(1 - k)] + \alpha \ln Gur_0 \rightarrow \max \qquad (13)$$

$$Gur_0 + G u(r - r_0)(1 - k) \leq VP; \qquad (14)$$

$$0 \leq r_0 \leq r ; \ \alpha, \beta, u \in (0,1) ; \ \alpha + \beta = 1; \ \alpha, \beta, k \geq 0 \qquad (15)$$

Here, (14) the constraint indicates that the conditionally economically the tax collected from the economic entity cannot be more than its tax potential. (15) indicates the necessary constraint conditions.

It should be noted that since the economic entity tends to remain in the shadow, its contribution to the total utility α, β varies depending on cost k remaining in the shadow.

5 Construction of IF − THEN fuzzy model

At the initial stage, as mentioned above, the economic entity declares a certain part (r_0) of its real profitability (r) and appropriates the remaining part (r - r_0). r is known to the tax authority at the level of the average value of the area to which the economic entity belongs. Therefore, in order to eliminate this uncertainty in the report of the economic entity, the tax authority will determine penalty of k_1 if $r_0 \in (r_{01}, r_{02})$ and if $r_0 \in (r_{02}, r_{03})$ it determines penalty of k_2, and if $r_0 \in (r_{03}, r)$, it determines penalty of k_3 and informs the economic entity about it. In this case, it is obvious that $k_1 > k_2 > k_3$. The economic entity declares a new greater value of r_0 in order not to lose its income, taking into account the applied penalty, and so on. Let's consider profitability r_0 of economic activity acting as a governing factor, cost k remaining in the shadow, contribution of taxes and income to the total utility and evaluation of α, β with IF-THEN fuzzy approach [7,8]. For this, based on the tax potential of the economic entity, taking into account the conditions (8) and (15) and accepting $\beta = k$ and $\alpha = 1 - \beta$ for simplicity, we shall present fuzzy approach for the selection of appropriate k:

- fuzzify k and r_0 using the existing fuzzy methods;
- determine the values of k and α with 3 linguistic quantities:
"weak", "medium", "strong"

    - also express the values of r_0 with three linguistic quantities in each interval $(r_{01}, r_{02,})$, (r_{02}, r_{03}) and (r_{03}, r):
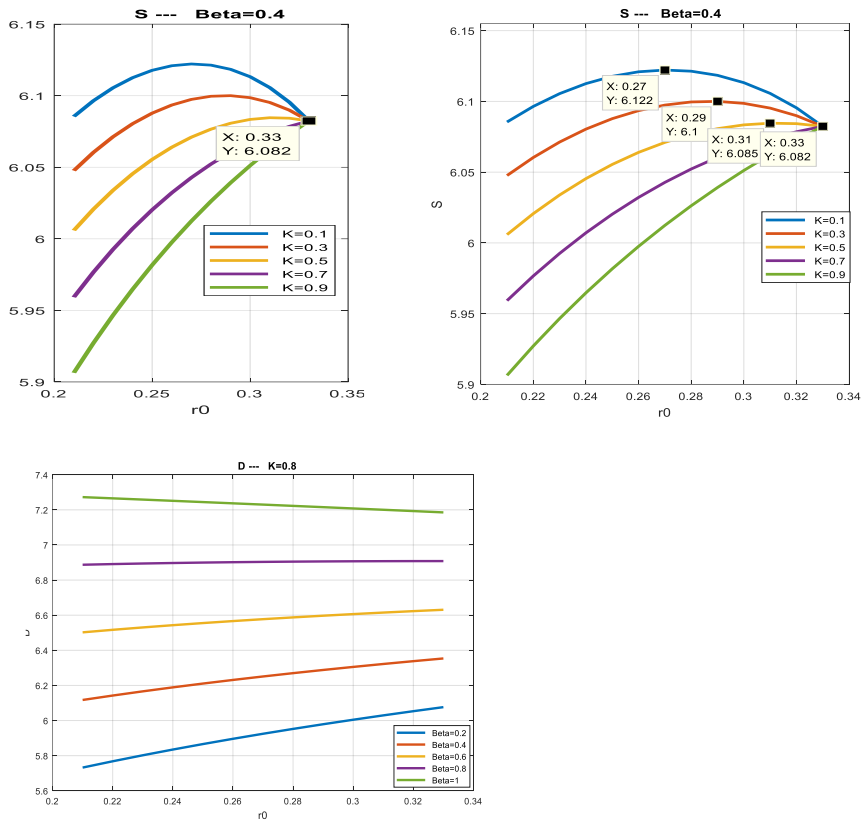
"low", "medium", "upper"

    According to these linguistic variables, we can write IF-THEN conditions as follows:

IF k = "weak" and α = "strong" Then r_0 = "upper"

IF k = "medium" and α = "medium" Then r_0 = "medium"

IF k = "weak" and α = "strong" Then r_0 = "upper"

IF k = "strong" and α = "weak" Then r_0 = "low"

    The fragment of the calculations of the proposed model and fuzzy approach on the sample will be as follows:

    Initial data: For example, G = 4000\$, P = 150\$, VP = 500\$,

u = 0.2,   r = 0.33, r_{min} = 0.21,   k = 0.2

    The increase in the value of the utility function of the state depending on the increase in profitability declared by the economic entity is shown in the graph below (see Fig. 1; Fig. 2a, 2b):
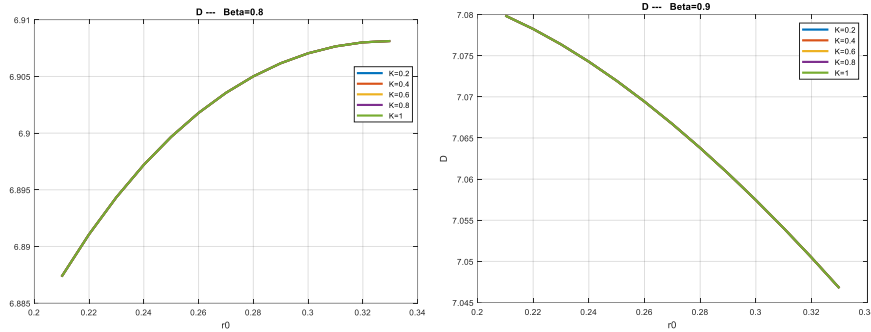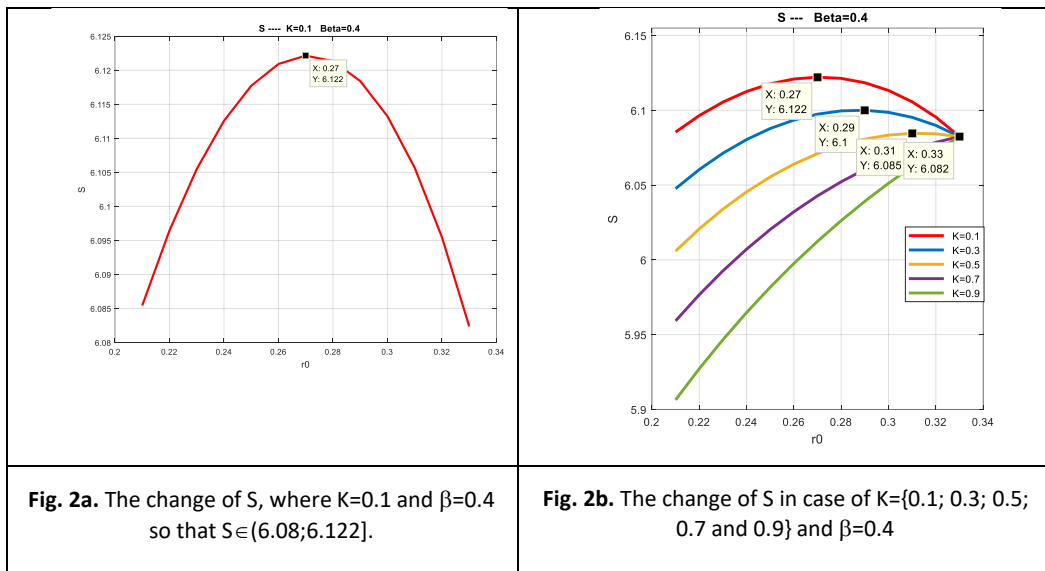
Fig. 1. In case of K={0.1; …;1} the change of D at different valuables of β

| **Fig. 2a.** The change of S, where K=0.1 and β=0.4 so that S∈(6.08;6.122]. | **Fig. 2b.** The change of S in case of K={0.1; 0.3; 0.5; 0.7 and 0.9} and β=0.4 |

Note: In Figures 2a and 2b ▬▬ The line represents the change in S at K = 0.1 and β= 0.4, the difference in convexity in the visual appearance of the graphs is due to the difference in scale on the ordinate axis.

6   Conclusion

— Shadow economy usually exists in areas where the tax burden is high, the state puts great pressure on business, and business transactions are possible to be realized;

— The obtained results show that the proposed instrumentation reflects the process of shadow economy emergence quite realistically;

— It is possible to increase the declared profitability (income) only by increasing the penalties, regardless of the utility of the collection;

— Declared profitability (income) can also be increased by increasing the profitability of the collection without changing the penalties;
— When analyzing these graphs, it is easy to observe that as the value of k penalty increases, the tendency of an economic entity to come out of the shadow increases;
— As the value of the declared profitability r_0 approaches the real profitability r, tax revenues to the state budget increase (Figure 1.);
— The economic entity is obliged to declare to the state a higher value of r_0 in order not to reduce the value of the utility function (Figure 2);
— At a certain value of shadow penalty k, the value of r_0 significantly approaches the real profitability r;
— As the value of k penalty increases, the shadow income decreases significantly.

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